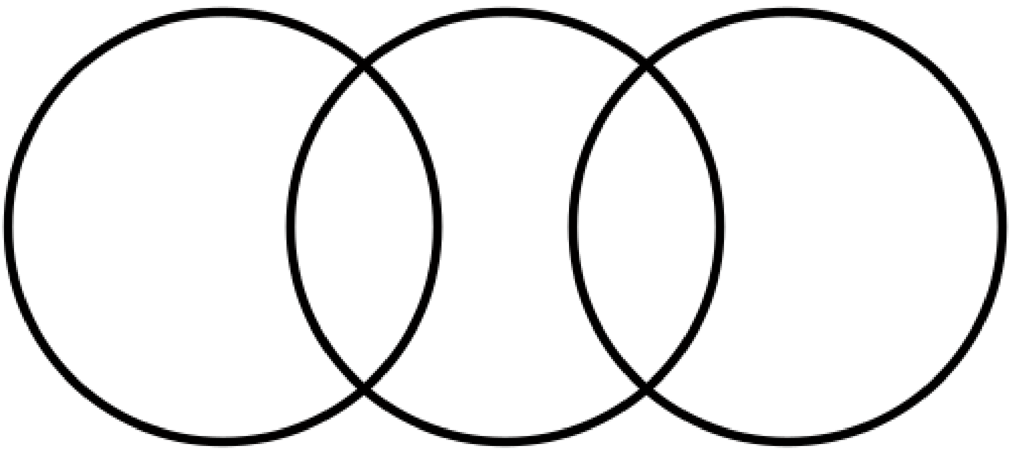
Puzzle of the Week

Equal Sums – 1

THE CHALLENGE: Here is a diagram created by overlapping three circles. The overlapping circles create five regions. Put a number in each of the five regions, using each of the numbers 1 to 5 exactly once, so that the sum of the numbers in each circle is the same.



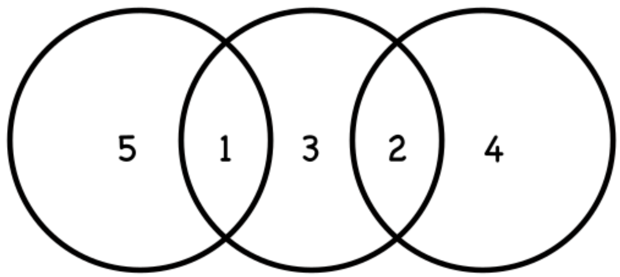
1 2 3 4 5

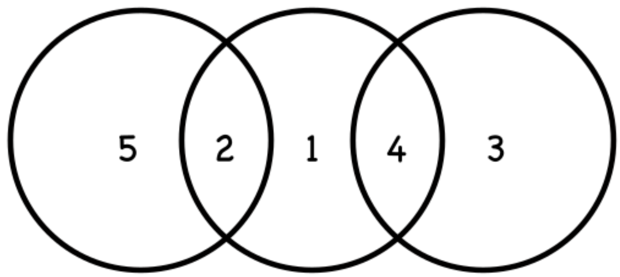
EXPLORATION: How many different answers can you find? How do you know if you have found them all?

Puzzle of the Week

Equal Sums – 1 – Notes

THE CHALLENGE & EXPLORATION: There are two solutions. Going from left to right, the sum in each circle is 6 and 7.





Analyze the possibilities by letting A and B be the two numbers in the intersections of the circles. Let Sum be the common sum inside each circle. Then 3 x Sum = 1 + 2 + 3 + 4 + 5 + A + B = 15 + A + B.

The left side of 3 x Sum = 15 + A + B is a multiple of 3, so the right side is as well. This forces A + B to be a multiple of three. That leaves only three possibilities.

- A + B = 3. In this case 3 x Sum = 15 + 3 = 18 tells us Sum = 6, and A and B are 1 and 2.
- A + B = 6. In this case 3 x Sum = 15 + 6 = 21 tells us Sum = 7. A + B = 6 forces A and B to be either 1 and 5 or 2 and 4. Having A and B be 1 and 5 does not work (1 is repeated), so that leaves us with just 2 and 4.
- A + B = 9. In this case 3 x Sum = 15 + 9 = 24 tells us Sum = 8, and A and B are 4 and 5. However, because A and B are both in the middle circle, it is not possible for A + B = 9 and yet the Sum is only 8. So this case cannot happen.